RIEŠENIE ROVNICE f(x)=0 NUMERICKÝMI METÓDAMI

Rieste rovnicu: **sin x = -3/(x+5)** *na intervale ⟨-3; 5,5⟩*

```
Clear[f]
f[x_] = 3 / (x + 5) + Sin[x]
```

$\frac{3}{5+x} + \mathrm{Sin}[x]$

```
Solve[f[x] == 0, x]
```

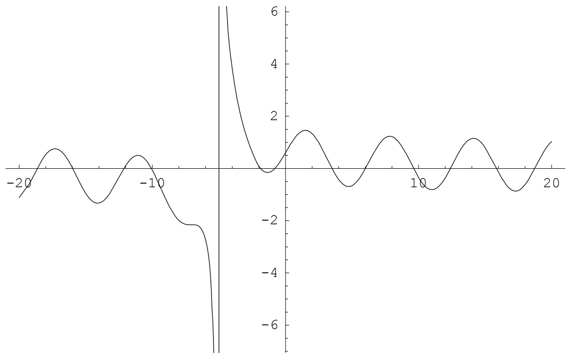
Solve::tdep : The equations appear to involve the
variables to be solved for in an essentially non-algebraic way. More…

$\mathrm{Solve}\left[\frac{3}{5+x} + \mathrm{Sin}[x] == 0,\ x\right]$

Rovnica sa nedá riešiť priamo, algebraickými metódami. Budeme ju riešiť numericky.

Graf funkcie na väčšom intervale:

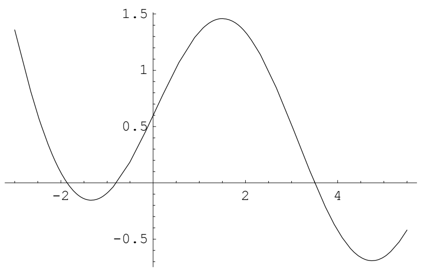
```
Plot[f[x], {x, -20, 20}]
```



- Graphics -

Graf funkcie na žiadanom intervale:

```
Plot[f[x], {x, -3, 5.5}]
```



- Graphics -

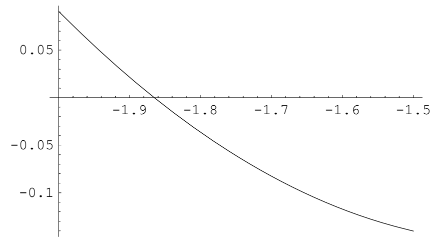
Riešenie rovnice numerickou metódou vyžaduje 3 kroky:
1. Separácia koreňov - určenie intervalov⟨a,b⟩, v ktorých sa nachádza práve jeden koreň, funkcia je spojitá, ostro monotónna, bez stacionárnych bodov (neobsahuje lokálne extrémy ani inflexné body)
2. Riešenie numerickou metódou (metóda sečníc -Secant, Newtonona metóda)
3. Určenie odhadu absolútnej chyby.

Metóda lineárnej interpolácie - všeobecná metóda tetív - SECANT $x_{i+1} = x_i - f(x_i)\,\frac{x_{i-1}-x_i}{f(x_{i-1})-f(x_i)}$

1. koreň

Separácia koreňa:

```
Plot[f[x], {x, -2, -1.5}]
```



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V inervale leží práve jeden koreň, interval je rýdzo-monotónny bez extrémov a inflexných bodov. Krajné body intervalu zvolíme ako štartovacie body iterácií.

```
x[0] = -2; x[1] = -1.5;
Do[
 x[i + 1] = x[i] - f[x[i]] * (x[i - 1] - x[i]) / (f[x[i - 1]] - f[x[i]]);
 Print["x(", i + 1, ")=", x[i + 1] // N],
 {i, 1, 10}]
```

x(2)=-1.80372

x(3)=-1.90235

x(4)=-1.8629

x(5)=-1.86508

x(6)=-1.86516

x(7)=-1.86516

x(8)=-1.86516

x(9)=-1.86516

x(10)=-1.86516

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. More…

```
∞::indet : Indeterminate expression 0. ComplexInfinity encountered. More…
```

```
x(11)=Indeterminate
```

Skopírujte na obrazovku výstup pre x(8), x(9) a x(10)

```
"x("8")="-1.8651605268487734`
"x("9")="-1.8651605268487736`
"x("10")="-1.8651605268487736`
```

```
x(8)=-1.86516

x(9)=-1.86516

x(10)=-1.86516
```

```
SetAccuracy[x[9], 15]
```

```
-1.865160526848774
```

```
SetAccuracy[x[10], 15]
```

```
-1.865160526848774
```

```
f[x[8]]
```

-1.11022×10^{-16}

```
f[x[9]]
```

```
0.
```

```
FindRoot[f[x], {x, -2, -1.5}]
```

```
{x → -1.86516}
```

```
1.8651605268487736`
```

```
1.86516
```

Riešte rovnicu: $\sin x = -3/(x+5)$ *na intervale* $\langle -3;\ 5,5\rangle$ **s toleranciou tol** $= 10^{-8}$, *keď* $|f(x_n)| \prec$ *tol.*

Cyklus s podmienkou: $|f(x_i)| <$ tolerancia *a rozšíreným výpisom.*

```
Clear[x]
x[0] = -2; x[1] = -1.5; tol = 10^(-8);
Do[
 x[i+1] = x[i] - f[x[i]] * (x[i-1] - x[i]) / (f[x[i-1]] - f[x[i]]) // N;
 Print["x(", i+1, ")=", x[i+1]];
 If[Abs[f[x[i+1]]] < tol,
               Print["|f(xn)|=  ", Abs[f[x[i+1]]]];
                Print["Počet iterácií:", i];
                Break[]],
 {i, 1, 10}]
```

```
x(2)=-1.80372

x(3)=-1.90235
```



```
x(4)=-1.8629

x(5)=-1.86508

x(6)=-1.86516

x(7)=-1.86516
```

$|f(x_n)| = 7.82974 \times 10^{-12}$

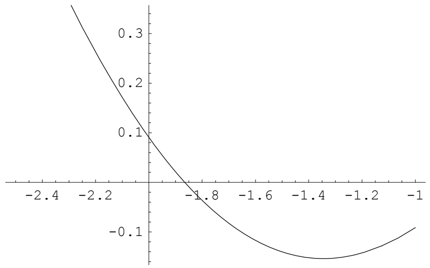
```
Počet iterácií:6
```

```
rS1 = x[7]
```

```
-1.86516
```

Pozor na vhodný interval !

```
Plot[f[x], {x, -2.5, -1}]
```




```
- Graphics -
```

```
x[0] = -2.5; x[1] = -1.2; tol = 10^(-8);
Do[
 x[i+1] = x[i] - f[x[i]] * (x[i-1] - x[i]) / (f[x[i-1]] - f[x[i]]);
 Print["x(", i+1, ")=", x[i+1] // N;
 If[Abs[f[x[i+1]]] < tol,
               Print["|f(xn)|=  ",         Abs[f[x[i+1]]]];
                Print["Pocet iteracii:", i];
                Break[]],
 {i, 1, 30}]
```

```
x(2)=-1.44907

x(3)=5.64795

x(4)=-7.84801

x(5)=8.06258

x(6)=2.16935

x(7)=202.512

x(8)=1052.41

x(9)=1074.19

x(10)=1054.56
```

```
x(11)=1081.5
x(12)=1069.15
x(13)=1149.06
x(14)=1113.31
x(15)=1133.87
x(16)=1141.14
x(17)=1135.81
x(18)=1152.26
x(19)=1145.76
x(20)=1180.22
x(21)=1162.45
x(22)=1163.69
x(23)=1162.36
x(24)=1162.39
x(25)=1162.39
x(26)=1162.39
```

$|f(x_n)| = \; 5.01506 \times 10^{-13}$

```
Pocet iteracii:25
```

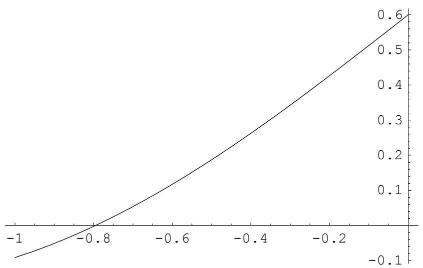
```
FindRoot[f[x], {x, -2.5, -1.2}]
```

$\{x \to -1.86516\}$

Metóda lineárnej interpolácie - všeobecná metóda tetív - SECANT
2. koreň

Separácia koreňa:

```
Plot[f[x], {x, -1, 0}]
```



- Graphics -

V inervale leží práve jeden koreň, interval je rýdzo-monotónny bez extrémov a inflexných bodov. Krajné body intervalu zvolíme ako štartovacie body iterácií.

```
a = -1; b = 0;
Clear[x]
x[0] = a; x[1] = b; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] * (x[i - 1] - x[i]) / (f[x[i - 1]] - f[x[i]]) // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
              Print["|f(xn)|=  ", Abs[f[x[i + 1]]]];
              Print["Počet iterácií:", i];
               Break[]],
 {i, 1, 10}]
```

```
x(2)=-0.867715368
x(3)=-0.8174900034
x(4)=-0.7927712897
x(5)=-0.7942207306
x(6)=-0.794195380
x(7)=-0.794195353
```

$|f(x_n)| = \; 2.71672 \times 10^{-13}$

```
Počet iterácií:6
```

```
rS2 = x[7]
```

-0.794195

Príkaz pre metódu Secant:

```
FindRoot[f[x], {x, a, b}]
```

$\{x \to -0.794195\}$

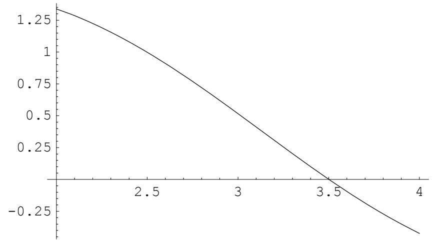
```
SetAccuracy[%, 10]
```

$\{x \to -0.794195353\}$

Metóda lineárnej interpolácie - všeobecná metóda tetív - SECANT
3. koreň

Separácia koreňa:

```
Plot[f[x], {x, 2, 4}]
```



- Graphics -

V inervale leží práve jeden koreň, interval je rýdzo-monotónny bez extrémov. Krajné body intervalu zvolíme ako štartovacie body iterácií.

```
a = 2; b = 4;
Clear[x]
x[0] = a; x[1] = b; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] * (x[i - 1] - x[i]) / (f[x[i - 1]] - f[x[i]]) // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
              Print["|f(xn)|=  ", Abs[f[x[i + 1]]]]];
              Print["Počet iterácií:", i];
                       Break[]],
 {i, 1, 10}]
```

x(2)=3.519150603

x(3)=3.499650638

x(4)=3.502215597

x(5)=3.502207442

$|f(x_n)| = 3.77202 \times 10^{-9}$

Počet iterácií:4

```
rS3 = x[5]
```

3.50221

Príkaz pre metódu Secant:

```
FindRoot[f[x], {x, a, b}]
```

$\{x \to 3.50221\}$

```
SetAccuracy[%, 10]
```

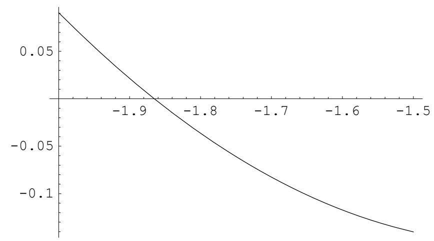

$\{x \to 3.502207439\}$

NEWTONOVA METÓDA $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

1. koreň

Separácia koreňa:

```
Plot[f[x], {x, -2, -1.5}]
```



- Graphics -

```
a = -2; b = -1.5;
Clear[x]
x[0] = a; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] / f'[x[i]] // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
              Print["|f(xn)|=  ", Abs[f[x[i + 1]]]]];
             Print["Pocet iteracii:", i + 1];
              Break[]],
 {i, 0, 10}]
```

x(1)=-1.8789793555

x(2)=-1.865340263

x(3)=-1.865160558

x(4)=-1.8651605268

$|f(x_n)| = 5.55112 \times 10^{-16}$

Pocet iteracii:4

```
rN1 = x[4]
```

-1.86516

Príkaz pre metódu Newton:

```
FindRoot[f[x], {x, a}]
```

$\{x \to -1.86516\}$

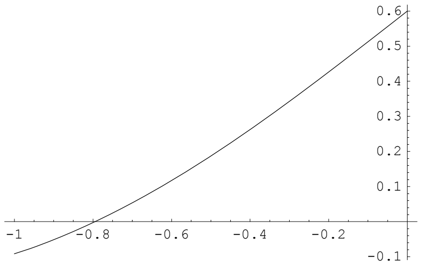
```
SetAccuracy[%, 10]
```

```
{x → -1.8651605268}
```

2. koreň

Separácia koreňa:

```
Plot[f[x], {x, -1, 0}]
```




```
- Graphics -
```

```
a = -1; b = 0;
Clear[x]
x[0] = a; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] / f'[x[i]] // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
               Print["|f(xn)|=  ", Abs[f[x[i + 1]]]]];
              Print["Počet iterácií:", i + 1];
               Break[]],
 {i, 0, 10}]
```

x(1)=-0.7407301957

x(2)=-0.792282241

x(3)=-0.794192629

x(4)=-0.794195353

$|f(x_n)| = \ 2.94509\times 10^{-12}$

Počet iterácií:4

Príkaz pre metódu Newton:

```
FindRoot[f[x], {x, a}]
```

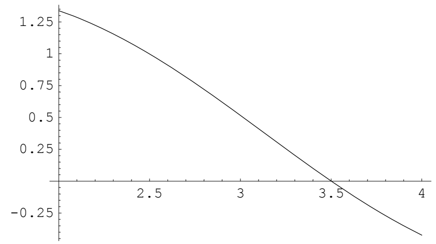
```
{x → -0.794195}
```

```
SetAccuracy[%, 10]
```

```
{x → -0.794195353}
```

3. koreň


```
Plot[f[x], {x, 2, 4}]
```




```
- Graphics -
```

Pozor na kontrolu A NEVHODNÉ BODY!

```
FindRoot[f''[x] == 0, {x, a, b}]
```

```
{x → 0.0466968}
```

V intervale leží INFLEXNÝ BOD!!

```
Clear[x]
x[0] = 2; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] / f'[x[i]] // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
               Print["|f(xn)|=  ", Abs[f[x[i + 1]]]]];
              Print["Pocet iteracii:", i + 1];
               Break[]],
 {i, 0, 10}]
```

x(1)=4.8025748125

x(2)=16.526881498

x(3)=15.669652848

x(4)=15.851947917

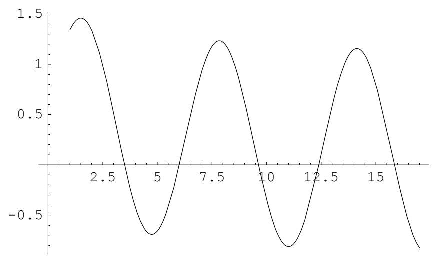
x(5)=15.852333036

x(6)=15.852333047

$|f(x_n)| = \ 6.93889\times 10^{-16}$

Pocet iteracii:6

```
Plot[f[x], {x, 1, 17}]
```




```
- Graphics -
```

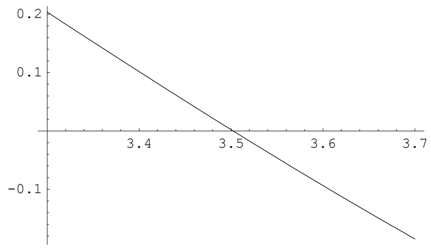
Príkaz pre metódu Newton:

```
FindRoot[f[x] == 0, {x, 2}]
```

```
{x → 15.8523}
```

Správna separácia koreňa:

```
a = 3.3; b = 3.7;
Plot[f[x], {x, a, b}]
```




```
- Graphics -
```

V inervale leží práve jeden koreň, interval je rýdzo-monotónny bez extrémov a inflexných bodov.

```
Clear[x]
x[0] = a; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] / f'[x[i]] // N;
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
               Print["|f(xn)|=  ", Abs[f[x[i + 1]]]];
              Print["Pocet iteracii:", i + 1];
               Break[]],
 {i, 0, 10}]
```

```
x(1)=3.497569995
x(2)=3.502203487
x(3)=3.502207439
```


$|f(x_n)| =$ 2.83135×10^{-12}

```
Pocet iteracii:3
```

```
FindRoot[f[x], {x, a}]
```

```
{x → 3.50221}
```

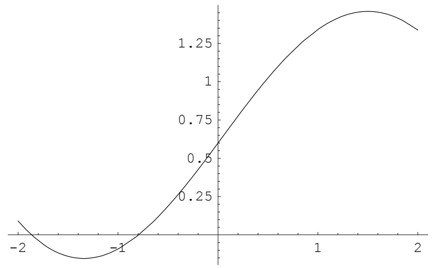
```
SetAccuracy[%, 10]
```

```
{x → 3.502207439}
```

Pozor na stacionárne body (extrém, infl. bod)! Kde $f'(x_0) = 0$ $f'(x_0)$ sa nachádza v menovateli - DELENIE NULOU !!!

Štartovací bod v blízkom okolí extrému
- delenie malým číslom - veľká chyba

```
Plot[f[x], {x, -2, 2}]
```




```
- Graphics -
```

```
Ex = FindRoot[f'[x] == 0, {x, -1.5}]
```

```
{x → -1.34438}
```

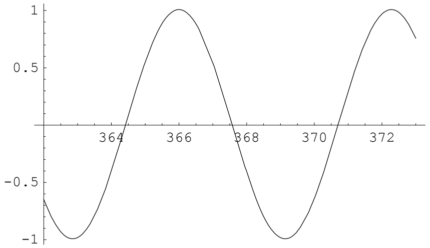
```
Clear[x]
x[0] = -1.344; tol = 10^(-8);
Do[
 x[i + 1] = x[i] - f[x[i]] / f'[x[i]];
 Print["x(", i + 1, ")=", SetAccuracy[x[i + 1], 10]];
 If[Abs[f[x[i + 1]]] < tol,
                Print["|f(xn)|=  ", Abs[f[x[i + 1]]] // N];
               Print["Pocet iteracii:", i + 1];
                Break[]],
 {i, 0, 10}]
```

```
x(1)=371.474052550
x(2)=370.500798466
x(3)=370.702787769
x(4)=370.699947968
x(5)=370.6999479431
```

|f(x_n)|= 3.33067×10^{-15}

Pocet iteracii:5

Plot[f[x], {x, 362, 373}]



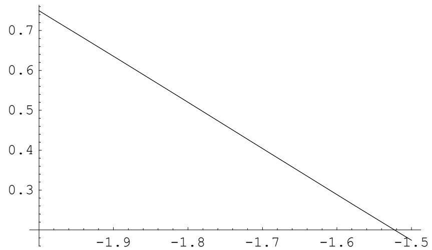
- Graphics -

FindRoot[f[x] == 0, {x, -1.344}]

{x → 22.1021}

Vypočítajte odhad abs. chyby:
na intervale <a, b> a= -2, b=-1.5 pre približnú hodnotu koreňa x_n = r1 = -1.8651605268487736

a = -2; b = -1.5;
Plot[Abs[f'[x]], {x, a, b}]



- Graphics -

m = Abs[f'[-1.5]]

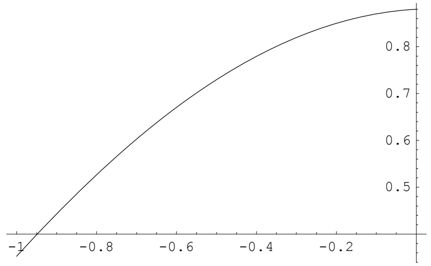
0.174161

eps = Abs[f[r1]] / m

5.74182 Abs$\left[\frac{3}{5+r1}+\text{Sin}[r1]\right]$

Odhad abs. chyby:
na intervale <a, b> a= -1, b=0 a koreň x_n = r2 = - 0.794195
eps = $\frac{f(x_n)}{m}$, m = min |$f'(x)$| on ⟨a, b⟩

Plot[Abs[f'[x]], {x, -1, 0}]



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r2

r2

m = Abs[f'[-1]] // N

0.352802

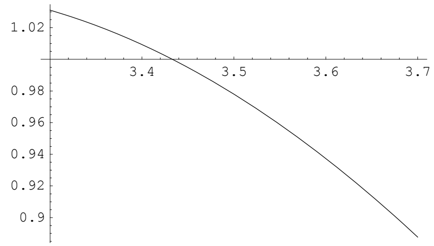
eps = Abs[f[r2]] / m // N

2.83445 Abs$\left[\frac{3.}{5.+r2}+\text{Sin}[r2]\right]$

Odhad abs. chyby:
na intervale <a, b> a=3.3 ; b=3.7 a koreň x_n = r3 =3.50221
eps = $\frac{f(x_n)}{m}$, m = min |$f'(x)$| on ⟨a, b⟩

a = 3.3; b = 3.7
Plot[Abs[f'[x]], {x, a, b}]

3.7



- Graphics -

r3

r3

m = Abs[f'[3.7]] // N

0.887735

```
eps = Abs[f[r3]] / m // N
```

$1.12646\,\mathrm{Abs}\left[\frac{3.}{5.+r3}+\mathrm{Sin}[r3]\right]$

SAMOSTATNÁ PRÁCA:

Riešte rovnicu $\ln x - \sin x = x - 2$ *na intervale* $\langle 0;\ 5\rangle$.

Úlohy:

1. *Koľko koreňov nadobúda rovnica na danom intervale?*
2. *Separujte najmenší (najväčší, druhý v poradí,....) z koreňov.*
3. *S toleranciou* tol $=10^{-7}$ *nájdite jeho približnú hodnotu* ($|f(x_n)| < tol$) *metódou lineárnej interpolácie (metódou tetív). Výsledok porovnajte s hodnotou koreňa nájdenou pomocou prikazu FindRoot s parametrami tejto metódy. Vypočítajte odhad absolútnej chyby.*
4. *S toleranciou* tol $=10^{-7}$ *nájdite jeho približnú hodnotu* ($|f(x_n)| < tol$) *Newtonovou metódou (metódou dotyčníc). Výsledok porovnajte s hodnotou koreňa nájdenou pomocou príkazu FindRoot s parametrami tejto metódy. Vypočítajte odhad absolútnej chyby.*
6. *Uveďte aspoň jeden bod, ktorý nie je vhodné použiť ako štartovací bod pre Newtonovu metodu Zdôvodnite.*
7. *Uveďte aspoň jednu dvojicu bodov, ktorú nie je vhodné použiť ako štartovacie body pre všeobecnú metódu tetív. Zdôvodnite.*