# Stabilita kritických bodov lineárnych sústav obyčajných diferenciálnych rovníc (SODR)

V tejto kapitole využijeme systém *Mathematica* k štúdiu stability kritických bodov dvoch lineárnych rovníc SODR.
Ak je SODR lineárny a homogénny $\boldsymbol{X}' = \boldsymbol{AX}$ s konštantnými koeficientmi a matica $\boldsymbol{A}$ nie je singulárna, potom jediným kritickým bodom je začiatok a jeho stabilitu môžeme preskúmať pomocou vlastných čísel matice $\boldsymbol{A}$.

## Príklad 1.

Porovnajte nasledujúci SODR, overte ich rozdielne správanie pomocou analýzy stability v kritickom bode:
*a)* $x' = 3x - y$, $y' = 2x + y$.
*b)* $x' = x + 5y$, $y' = -x - y$.
c) $x' = x - 5y$, $y' = -x - y$.

## Riešenie

*Poznámka*: Tieto tri sústavy sú autonómne, lineárne s konštantnými koeficientmi a s jediným kritickým bodom v začiatku súradnicovej sústavy.
Niektoré trajektórie je možné reprezentovať pomocou fázových rovín.

## ■ Sekcia a.

SODR môžeme riešiť v Mathematice pomocou príkazu **DSolve**

```
sis31a = {x'[t] == 3 x[t] - y[t], y'[t] == 2 x[t] + y[t]}
```

```
{x′[t] == 3 x[t] - y[t], y′[t] == 2 x[t] + y[t]}
```

```
solgen31a = DSolve[sis31a, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to -e^{2t}\, C[2]\, \mathrm{Sin}[t] + e^{2t}\, C[1]\, (\mathrm{Cos}[t] + \mathrm{Sin}[t]),\ y[t] \to e^{2t}\, C[2]\, (\mathrm{Cos}[t] - \mathrm{Sin}[t]) + 2\, e^{2t}\, C[1]\, \mathrm{Sin}[t]\right\}\right\}$$

Riešenia *x(t)* a *y(t)* sú definované:

```
x[t_] = solgen31a[[1, 1, 2]]
```

$-e^{2t}$ C[2] Sin[t] + $e^{2t}$ C[1] (Cos[t] + Sin[t])

```
y[t_] = solgen31a[[1, 2, 2]]
```

$e^{2t}$ C[2] (Cos[t] − Sin[t]) + 2 $e^{2t}$ C[1] Sin[t]

Pre niektoré hodnoty konštánt môžeme konkrétne riešenia vygenerovať pomocou príkazu **Table**

```
solpar31a = Table[{x[t], y[t]} /. {C[1] -> i, C[2] -> j}, {i, -6, 6, 4}, {j, -6, 6, 4}]
```

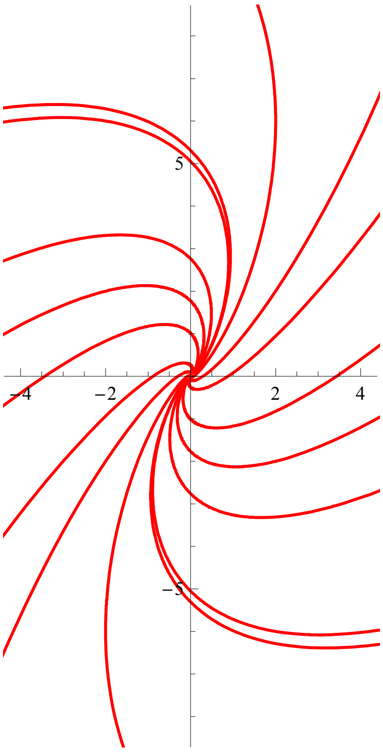
```
{{{6 e^(2 t) Sin[t] - 6 e^(2 t) (Cos[t] + Sin[t]), -6 e^(2 t) (Cos[t] - Sin[t]) - 12 e^(2 t) Sin[t]},
  {2 e^(2 t) Sin[t] - 6 e^(2 t) (Cos[t] + Sin[t]), -2 e^(2 t) (Cos[t] - Sin[t]) - 12 e^(2 t) Sin[t]},
  {-2 e^(2 t) Sin[t] - 6 e^(2 t) (Cos[t] + Sin[t]), 2 e^(2 t) (Cos[t] - Sin[t]) - 12 e^(2 t) Sin[t]},
  {-6 e^(2 t) Sin[t] - 6 e^(2 t) (Cos[t] + Sin[t]), 6 e^(2 t) (Cos[t] - Sin[t]) - 12 e^(2 t) Sin[t]}},
 {{6 e^(2 t) Sin[t] - 2 e^(2 t) (Cos[t] + Sin[t]), -6 e^(2 t) (Cos[t] - Sin[t]) - 4 e^(2 t) Sin[t]},
  {2 e^(2 t) Sin[t] - 2 e^(2 t) (Cos[t] + Sin[t]), -2 e^(2 t) (Cos[t] - Sin[t]) - 4 e^(2 t) Sin[t]},
  {-2 e^(2 t) Sin[t] - 2 e^(2 t) (Cos[t] + Sin[t]), 2 e^(2 t) (Cos[t] - Sin[t]) - 4 e^(2 t) Sin[t]},
  {-6 e^(2 t) Sin[t] - 2 e^(2 t) (Cos[t] + Sin[t]), 6 e^(2 t) (Cos[t] - Sin[t]) - 4 e^(2 t) Sin[t]}},
 {{6 e^(2 t) Sin[t] + 2 e^(2 t) (Cos[t] + Sin[t]), -6 e^(2 t) (Cos[t] - Sin[t]) + 4 e^(2 t) Sin[t]},
  {2 e^(2 t) Sin[t] + 2 e^(2 t) (Cos[t] + Sin[t]), -2 e^(2 t) (Cos[t] - Sin[t]) + 4 e^(2 t) Sin[t]},
  {-2 e^(2 t) Sin[t] + 2 e^(2 t) (Cos[t] + Sin[t]), 2 e^(2 t) (Cos[t] - Sin[t]) + 4 e^(2 t) Sin[t]},
  {-6 e^(2 t) Sin[t] + 2 e^(2 t) (Cos[t] + Sin[t]), 6 e^(2 t) (Cos[t] - Sin[t]) + 4 e^(2 t) Sin[t]}},
 {{6 e^(2 t) Sin[t] + 6 e^(2 t) (Cos[t] + Sin[t]), -6 e^(2 t) (Cos[t] - Sin[t]) + 12 e^(2 t) Sin[t]},
  {2 e^(2 t) Sin[t] + 6 e^(2 t) (Cos[t] + Sin[t]), -2 e^(2 t) (Cos[t] - Sin[t]) + 12 e^(2 t) Sin[t]},
  {-2 e^(2 t) Sin[t] + 6 e^(2 t) (Cos[t] + Sin[t]), 2 e^(2 t) (Cos[t] - Sin[t]) + 12 e^(2 t) Sin[t]},
  {-6 e^(2 t) Sin[t] + 6 e^(2 t) (Cos[t] + Sin[t]), 6 e^(2 t) (Cos[t] - Sin[t]) + 12 e^(2 t) Sin[t]}}}
```

Ak v predchádzajúcej tabuľke odstránime jednu úroveň pomocou príkazu **Flatten**, vytvoríme tak zoznam funkcií, ktorý je možné reprezentovať graficky.

```
solpar31abis = Flatten[solpar31a, 1]
```

$$\{\{6\,e^{2t}\,\text{Sin}[t]-6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{2\,e^{2t}\,\text{Sin}[t]-6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-2\,e^{2t}\,\text{Sin}[t]-6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-6\,e^{2t}\,\text{Sin}[t]-6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{6\,e^{2t}\,\text{Sin}[t]-2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{2\,e^{2t}\,\text{Sin}[t]-2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-2\,e^{2t}\,\text{Sin}[t]-2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-6\,e^{2t}\,\text{Sin}[t]-2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])-4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{6\,e^{2t}\,\text{Sin}[t]+2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{2\,e^{2t}\,\text{Sin}[t]+2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-2\,e^{2t}\,\text{Sin}[t]+2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-6\,e^{2t}\,\text{Sin}[t]+2\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+4\,e^{2t}\,\text{Sin}[t]\},$$
$$\{6\,e^{2t}\,\text{Sin}[t]+6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{2\,e^{2t}\,\text{Sin}[t]+6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ -2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-2\,e^{2t}\,\text{Sin}[t]+6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 2\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+12\,e^{2t}\,\text{Sin}[t]\},$$
$$\{-6\,e^{2t}\,\text{Sin}[t]+6\,e^{2t}\,(\text{Cos}[t]+\text{Sin}[t]),\ 6\,e^{2t}\,(\text{Cos}[t]-\text{Sin}[t])+12\,e^{2t}\,\text{Sin}[t]\}\}$$

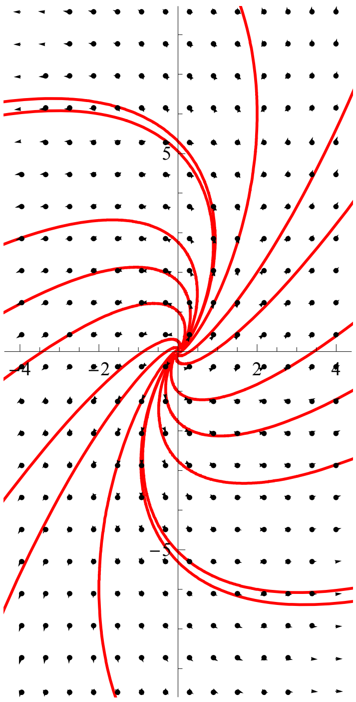
```
graf131a = ParametricPlot[Evaluate[solpar31abis],
  {t, -2, 0.5`}, PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.008`]}}]
```



V nasledujúcom obrázku sme spoločne reprezentovali riešenie a smerové polia riešenia SODR.

```
graf231a = (Needs["VectorFieldPlots`"];
   VectorFieldPlots`VectorFieldPlot[{3 x - y, 2 x + y}, {x, -15, 15}, {y, -20, 20},
    PlotPoints → 50, DisplayFunction → Identity, BaseStyle → {GrayLevel[0.2`]}]);
```

```
Show[graf131a, graf231a]
```




Trajektórie sa vzďaľujú od začiatku súradnicovej sústavy (zatáčajú sa do tvaru špirály), ak zväčšujeme *t*, a opačne sú blízko k začiatku súradnicovej sústavy, ak je *t* menšie ako 0. Toto správanie sa dá vysvetliť po podrobnejšom preskúmaní matice koeficientov SODR.

Matica asociovaná so sústavou je definovaná:

$$a = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$$

```
{{3, -1}, {2, 1}}
```

Jej vlastné čísla nájdeme takto:

```
Eigenvalues[a]
```

```
{2 + i, 2 - i}
```

Začiatok je nestabilný pól špirály, pretože vlastné čísla sú komplexne združené a majú kladnú reálnu časť.

## Sekcia b.

Najskôr vyčistíme obsah premenných a definujeme nový SODR

```
Clear[x, y]
```

```
sis31b = {x'[t] == x[t] + 5 y[t], y'[t] == -x[t] - y[t]}
```

```
{x′[t] == x[t] + 5 y[t], y′[t] == -x[t] - y[t]}
```

Túto sústavu môžeme vyriešiť pomocou príkazu **DSolve**:

```
solgen31b = DSolve[sis31b, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{5}{2} C[2] \operatorname{Sin}[2 t] + \frac{1}{2} C[1] (2 \operatorname{Cos}[2 t] + \operatorname{Sin}[2 t]), y[t] \to \frac{1}{2} C[2] (2 \operatorname{Cos}[2 t] - \operatorname{Sin}[2 t]) - \frac{1}{2} C[1] \operatorname{Sin}[2 t]\right\}\right\}$$

Definujeme riešenie *x(t)* a *y(t)*:

```
x[t_] = solgen31b[[1, 1, 2]]
```

$$\frac{5}{2} C[2] \operatorname{Sin}[2 t] + \frac{1}{2} C[1] (2 \operatorname{Cos}[2 t] + \operatorname{Sin}[2 t])$$

```
y[t_] = solgen31b[[1, 2, 2]]
```

$$\frac{1}{2} C[2] (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - \frac{1}{2} C[1] \mathrm{Sin}[2 t]$$

Riešenia pre konkrétne hodnoty konštánt $c_1$ a $c_2$ vygenerujeme pomocou príkazu **Table**:

```
solpar31b = Table[{x[t], y[t]} /. {C[1] -> i, C[2] -> j}, {i, 2, 8, 3}, {j, 2, 8, 3}]
```

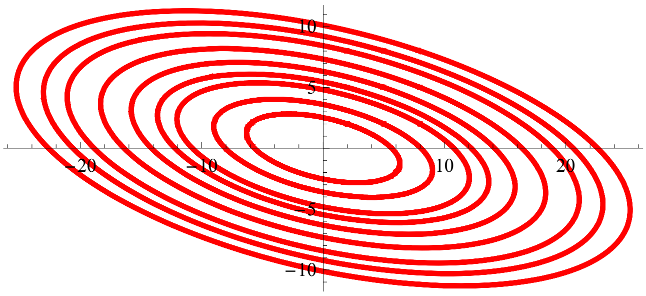
$$\Big\{\Big\{\{2 \mathrm{Cos}[2 t] + 6 \mathrm{Sin}[2 t], 2 \mathrm{Cos}[2 t] - 2 \mathrm{Sin}[2 t]\},$$
$$\Big\{2 \mathrm{Cos}[2 t] + \frac{27}{2} \mathrm{Sin}[2 t], \frac{5}{2} (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - \mathrm{Sin}[2 t]\Big\},$$
$$\{2 \mathrm{Cos}[2 t] + 21 \mathrm{Sin}[2 t], 4 (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - \mathrm{Sin}[2 t]\}\Big\},$$
$$\Big\{\Big\{5 \mathrm{Sin}[2 t] + \frac{5}{2} (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), 2 \mathrm{Cos}[2 t] - \frac{7}{2} \mathrm{Sin}[2 t]\Big\},$$
$$\Big\{\frac{25}{2} \mathrm{Sin}[2 t] + \frac{5}{2} (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), \frac{5}{2} (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - \frac{5}{2} \mathrm{Sin}[2 t]\Big\},$$
$$\Big\{20 \mathrm{Sin}[2 t] + \frac{5}{2} (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), 4 (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - \frac{5}{2} \mathrm{Sin}[2 t]\Big\}\Big\},$$
$$\Big\{\{5 \mathrm{Sin}[2 t] + 4 (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), 2 \mathrm{Cos}[2 t] - 5 \mathrm{Sin}[2 t]\},$$
$$\Big\{\frac{25}{2} \mathrm{Sin}[2 t] + 4 (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), \frac{5}{2} (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - 4 \mathrm{Sin}[2 t]\Big\},$$
$$\{20 \mathrm{Sin}[2 t] + 4 (2 \mathrm{Cos}[2 t] + \mathrm{Sin}[2 t]), 4 (2 \mathrm{Cos}[2 t] - \mathrm{Sin}[2 t]) - 4 \mathrm{Sin}[2 t]\}\Big\}\Big\}$$

Štruktúru predchádzajúcej tabuľky upravíme pomocou príkazu **Flatten**, aby sme vytvorili zoznam funkcií, ktorý môžeme reprezentovať graficky.

```
solpar31bbis = Flatten[solpar31b, 1]
```

$$\Big\{\{2\,\text{Cos}[2\,t]+6\,\text{Sin}[2\,t],\ 2\,\text{Cos}[2\,t]-2\,\text{Sin}[2\,t]\},$$
$$\Big\{2\,\text{Cos}[2\,t]+\frac{27}{2}\,\text{Sin}[2\,t],\ \frac{5}{2}\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-\text{Sin}[2\,t]\Big\},$$
$$\{2\,\text{Cos}[2\,t]+21\,\text{Sin}[2\,t],\ 4\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-\text{Sin}[2\,t]\},$$
$$\Big\{5\,\text{Sin}[2\,t]+\frac{5}{2}\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ 2\,\text{Cos}[2\,t]-\frac{7}{2}\,\text{Sin}[2\,t]\Big\},$$
$$\Big\{\frac{25}{2}\,\text{Sin}[2\,t]+\frac{5}{2}\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ \frac{5}{2}\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-\frac{5}{2}\,\text{Sin}[2\,t]\Big\},$$
$$\Big\{20\,\text{Sin}[2\,t]+\frac{5}{2}\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ 4\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-\frac{5}{2}\,\text{Sin}[2\,t]\Big\},$$
$$\{5\,\text{Sin}[2\,t]+4\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ 2\,\text{Cos}[2\,t]-5\,\text{Sin}[2\,t]\},$$
$$\Big\{\frac{25}{2}\,\text{Sin}[2\,t]+4\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ \frac{5}{2}\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-4\,\text{Sin}[2\,t]\Big\},$$
$$\{20\,\text{Sin}[2\,t]+4\,(2\,\text{Cos}[2\,t]+\text{Sin}[2\,t]),\ 4\,(2\,\text{Cos}[2\,t]-\text{Sin}[2\,t])-4\,\text{Sin}[2\,t]\}\Big\}$$

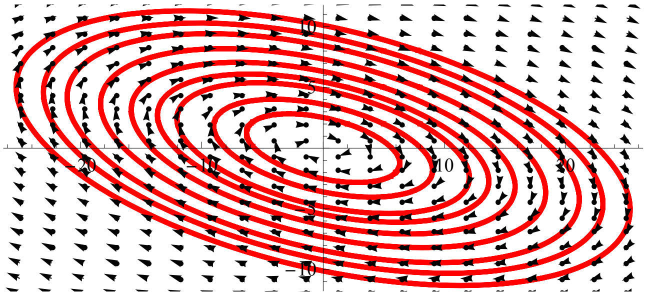
```
graf131b = ParametricPlot[Evaluate[solpar31bbis],
  {t, 0, 2 π}, PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.008`]}}]
```




V nasledujúcom obrázku sme spoločne reprezentovali riešenie a smerové polia riešenia SODR.

```
graf231b = (Needs["VectorFieldPlots`"];
   VectorFieldPlots`VectorFieldPlot[{x + 5 y, -x - y}, {x, -25, 25}, {y, -12, 12},
    PlotPoints → 20, DisplayFunction → Identity, BaseStyle → {GrayLevel[0.2`]}]);
```

```
Show[graf131b, graf231b]
```




Trajektórie sú uzatvorené krivky, v tomto prípade elipsy so stredom v začiatku súradnicovej sústavy. Toto správanie môžeme znovu opísať pomocou vlastných čísel matice koeficientov.

Matica zodpovedajúca našej sústave je definovaná:

$$a = \begin{pmatrix} 1 & 5 \\ -1 & -1 \end{pmatrix}$$

```
{{1, 5}, {-1, -1}}
```

jej vlastné čísla nájdeme pomocou príkazu:

```
Eigenvalues[a]
```

```
{2 i, -2 i}
```

Začiatok súradnicovej sústavy je stabilným stredom, pretože vlastné čísla sú rýdzo imaginárne.

## ■ Sekcia c.

Najskôr vyčistíme obsah premenných a definujeme nový SODR

```
Clear[x, y]
```

```
sis31c = {x'[t] == x[t] - 5 y[t], y'[t] == -x[t] - y[t]}
```

$\{x'[t] == x[t] - 5\,y[t],\ y'[t] == -x[t] - y[t]\}$

Vyriešime túto sústavu

```
solgen31c = DSolve[sis31c, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{1}{12} e^{-\sqrt{6}\,t}\left(6 - \sqrt{6} + 6\,e^{2\sqrt{6}\,t} + \sqrt{6}\,e^{2\sqrt{6}\,t}\right) C[1] - \frac{5\,e^{-\sqrt{6}\,t}\left(-1 + e^{2\sqrt{6}\,t}\right) C[2]}{2\sqrt{6}},\right.\right.$$
$$\left.\left. y[t] \to -\frac{e^{-\sqrt{6}\,t}\left(-1 + e^{2\sqrt{6}\,t}\right) C[1]}{2\sqrt{6}} - \frac{1}{12} e^{-\sqrt{6}\,t}\left(-6 - \sqrt{6} - 6\,e^{2\sqrt{6}\,t} + \sqrt{6}\,e^{2\sqrt{6}\,t}\right) C[2]\right\}\right\}$$

Riešenia *x(t)* a *y(t)* sú definované:

```
x[t_] = solgen31c[[1, 1, 2]]
```

$$\frac{1}{12} e^{-\sqrt{6}\,t}\left(6 - \sqrt{6} + 6\,e^{2\sqrt{6}\,t} + \sqrt{6}\,e^{2\sqrt{6}\,t}\right) C[1] - \frac{5\,e^{-\sqrt{6}\,t}\left(-1 + e^{2\sqrt{6}\,t}\right) C[2]}{2\sqrt{6}}$$

```
y[t_] = solgen31c[[1, 2, 2]]
```

$$-\frac{e^{-\sqrt{6}\,t}\left(-1 + e^{2\sqrt{6}\,t}\right) C[1]}{2\sqrt{6}} - \frac{1}{12} e^{-\sqrt{6}\,t}\left(-6 - \sqrt{6} - 6\,e^{2\sqrt{6}\,t} + \sqrt{6}\,e^{2\sqrt{6}\,t}\right) C[2]$$

Riešenia pre konkrétne hodnoty konštánt $c_1$ a $c_2$ vygenerujeme pomocou príkazu **Table**:

```
solpar31c =
 Simplify[Table[{x[t], y[t]} /. {C[1] -> i, C[2] -> j}, {i, 1, 5, 2}, {j, 1, 5, 2}]]
```

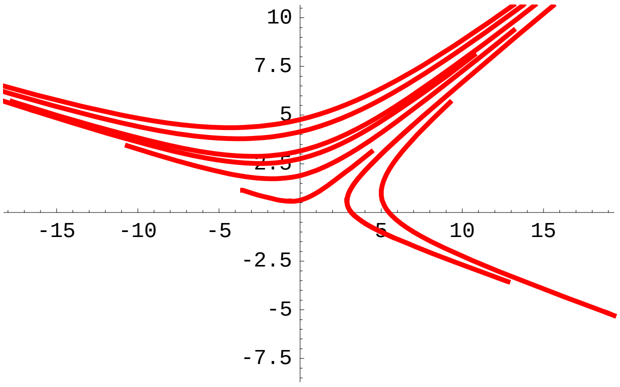
$$\left\{\left\{\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},\right.\right.$$
$$\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(3+7\sqrt{6}+\left(3-7\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(9+2\sqrt{6}+\left(9-2\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left.\left\{\frac{1}{2}e^{-\sqrt{6}t}\left(1+4\sqrt{6}+\left(1-4\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(5+\sqrt{6}-\left(-5+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\}\right\},$$
$$\left\{\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(9+\sqrt{6}-\left(-9+\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},\right.$$
$$\left\{\frac{1}{2}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left.\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(9+11\sqrt{6}+\left(9-11\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(15+4\sqrt{6}+\left(15-4\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\}\right\},$$
$$\left\{\left\{\frac{5}{2}e^{-\sqrt{6}t}\left(1+e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(1+\sqrt{6}-\left(-1+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},\right.$$
$$\left\{\frac{5}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(9+4\sqrt{6}+\left(9-4\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left.\left.\left\{\frac{5}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{5}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\}\right\}\right\}$$

Štruktúru predchádzajúcej tabuľky upravíme pomocou príkazu **Flatten**, aby sme vytvorili zoznam funkcií, ktorý môžeme reprezentovať graficky.

```
solpar31cbis = Flatten[solpar31c, 1]
```

$$\left\{\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},\right.$$
$$\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(3+7\sqrt{6}+\left(3-7\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(9+2\sqrt{6}+\left(9-2\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{1}{2}e^{-\sqrt{6}t}\left(1+4\sqrt{6}+\left(1-4\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(5+\sqrt{6}-\left(-5+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(9+\sqrt{6}-\left(-9+\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{1}{2}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{1}{6}e^{-\sqrt{6}t}\left(9+11\sqrt{6}+\left(9-11\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(15+4\sqrt{6}+\left(15-4\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{5}{2}e^{-\sqrt{6}t}\left(1+e^{2\sqrt{6}t}\right),\ \frac{1}{2}e^{-\sqrt{6}t}\left(1+\sqrt{6}-\left(-1+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left\{\frac{5}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{1}{6}e^{-\sqrt{6}t}\left(9+4\sqrt{6}+\left(9-4\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\},$$
$$\left.\left\{\frac{5}{6}e^{-\sqrt{6}t}\left(3+2\sqrt{6}+\left(3-2\sqrt{6}\right)e^{2\sqrt{6}t}\right),\ \frac{5}{6}e^{-\sqrt{6}t}\left(3+\sqrt{6}-\left(-3+\sqrt{6}\right)e^{2\sqrt{6}t}\right)\right\}\right\}$$

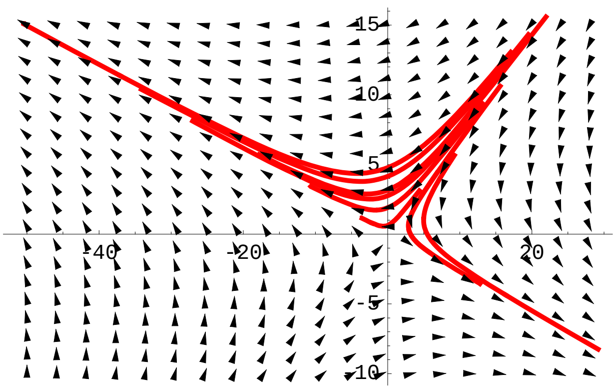
```
graf131c = ParametricPlot[Evaluate[solpar31cbis],
  {t, -0.5`, 1}, PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.008`]}}]
```




V nasledujúcom obrázku sme spoločne reprezentovali riešenie a smerové polia riešenia SODR.

```
graf231c = (Needs["VectorFieldPlots`"];
   VectorFieldPlots`VectorFieldPlot[{x - 5 y, -x - y}, {x, -50, 28}, {y, -10, 15},
    PlotPoints → 20, DisplayFunction → Identity, BaseStyle → {GrayLevel[0.2`]}]);
```

```
Show[graf131c, graf231c]
```




Vypočítame vlastné čísla matice zodpovedajúce našej sústave, aby sme mohli vysvetliť povahu kritického bodu.

Matica zodpovedajúca SODR je:

```
a = ( 1  -5
     -1  -1 )
```

```
{{1, -5}, {-1, -1}}
```

Vypočítame vlastné čísla:

```
Eigenvalues[a]
```

$\{-\sqrt{6}, \sqrt{6}\}$

Začiatok súradnicovej sústavy je sedlový bod, pretože vlastné čísla sú reálne a majú opačné znamienka.

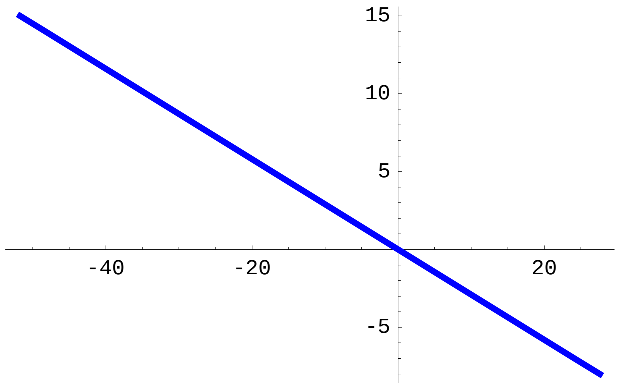
Vypočítame vlastné vektory, ktorými sú reprezentované priamky v zodpovedajúcich smeroch.

```
Eigenvectors[a]
```

$\{\{-1+\sqrt{6}, 1\}, \{-1-\sqrt{6}, 1\}\}$

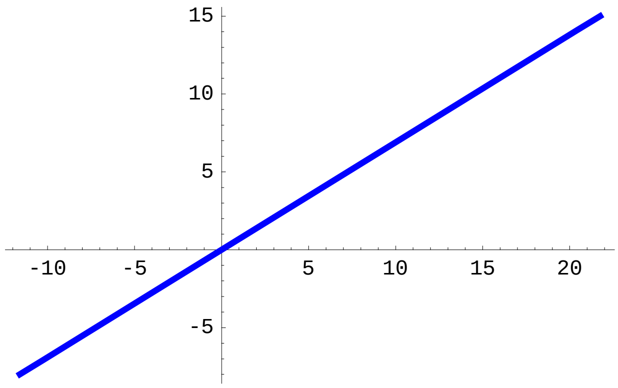
Priamka vlastného vektora, ktorý zodpovedá zápornému vlastnému číslu je

```
graf331c = ParametricPlot[{(-1 - √6) t, t},
  {t, -8, 15}, PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01`]}]
```



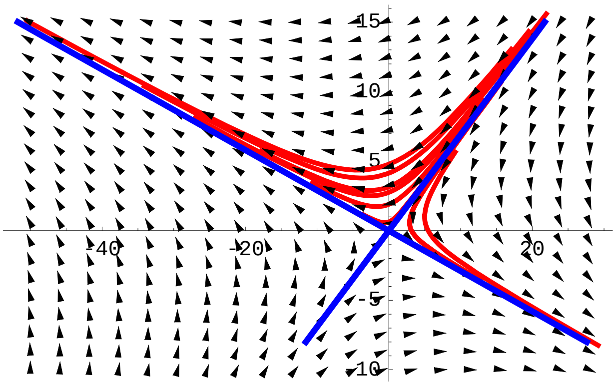

Priamka vlastného vektora, ktorý zodpovedá kladnému vlastnému číslu je

```
graf431c = ParametricPlot[{(-1 + √6) t, t},
  {t, -8, 15}, PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01`]}]
```




a oba grafy spolu:

```
Show[graf131c, graf231c, graf331c, graf431c]
```




Môžeme povedať, že akákoľvek trajektória začínajúca v bode na priamke, ktorá zodpovedá zápornému vlastnému číslu, zostává nad ňou a blíži sa k začiatku súradnicovej sústavy. Každá trajektória začínajúca v bode na priamke, ktorá zodpovedá kladnému vlastnému číslu, zostáva pod touto priamkou a tiež sa blíži k začiatku súradnicovej sústavy.